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VISCOSITY — SPECIFIC VOLUME ANOMALIES, A NOMOGRAM

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ABSTRACT

Viscosity anomaly is conceived and linked up with specific volume anomaly of sea water. A nomogram is prepared comprising the families of curves of isohalines and isotherms in the diagram having viscosity anomaly and specific volume anomaly at its coordinates of reference axes (rectangular coordinates). The nomogram is useful for studies of physical characteristics especially dynamic characteristics of sea water.

Key-words : Viscosity, specific volume, nomogram.

INTRODUCTION

Viscosity i.e. internal frictional force of a fluid moving in parallel layer is the shearing stress at any point within the fluid and is proportional to the velocity gradient perpendicular to the direction of motion. This is known as the Newton's law of viscosity for laminar flow. It may be expressed as

$$f = \eta \frac{\partial u}{\partial z} \quad (1)$$

where f is the shearing stress (tangential force/unit area), u is the velocity, $\frac{\partial u}{\partial z}$ is the velocity gradient at rightangles to the direction of u and η is the coefficient of dynamic viscosity of the fluid (Newman and Searle, 1951). The ratio of dynamic viscosity to the density of the fluid is termed as kinematic viscosity of the fluid. Sverdrup, Johnson and Fleming (1961) presented the results of Dorsey on the values of dynamic viscosity of sea water at various temperatures and salinities. The present study is based on those values. A nomogram of viscosity and specific volume anomalies conceived in the following lines is felt to reveal greater details of seawater characteristics.

Methodology

The density *in situ* hence the specific volume and viscosity are the functions of temperature, salinity and pressure. The effect of pressure on the above parameters is so small that it can be practically neglected (Sverdrup, Johnson and Fleming 1961, Newman and Searle, 1951). In the usual manner, treating specific volume at salinity 35‰ and temperature 0°C as standard, the anomalies of specific volume at salinity,s, and temperature,t, are expressed. If $\alpha_{s,t}$ and $\alpha_{35,0}$ represent the specific volumes, the former at salinity S and temperature t and the latter at S = 35‰ and t = 0 °C, then the specific volume anomaly is

$$\triangle_{s,t} = \alpha_{s,t} - \alpha_{35\cdot 0} \quad (2)$$

LaFond (1951) prepared the tables of specific volume anomaly at various temperatures and salinities. The specific volume anomaly and density are related as

$$\Delta_{s,t} = 0.273569 - \frac{10^{-3}\sigma_t}{1+10^{-3}\sigma_t} \tag{3}$$

In analogy to specific volume anomaly, consider the viscosity at S = 35‰ and t – 0 °C as standard. Thus,

$$\Delta\gamma_{s,t} = \gamma_{s,t} \quad \gamma_{35,0} \tag{4}$$

where $\Delta\gamma_{s,t}$ is the viscosity anomaly from $\gamma_{35,0}$ (= 18.9×10^{-3} C.G.S. units, Sverdrup op, cit). In the light of the above definition of $\Delta\gamma_{s,t}$, the sets of values of $\Delta\gamma_{s,t}$ and $\Delta_{s,t}$ corresponding to fixed values of S (=10,20,30 & 35‰) are examined graphically under different conditions of temperature. Table I presents the values of $\Delta_{s,t}$ and the corresponding $\Delta\gamma_{s,t}$ values for different steps of salinity and temperature values. In all the sets, the maximum value attainable by $-10^3\Delta\gamma_{s,t}$'s slightly more than 10 and it did not exceed 11. $10^5\Delta_{s,t}$ ranged from zero to about 2500 CGS units for the ranges of salinity from 10 to 35‰ and for the temperature range of 0°C to 30°C. The The curves are drawn showing the graphical relation between $\Delta\gamma_{s,t}$ and

Table I. The anomalies of viscosity and specific volume at different sets of temperature and salinity values.

t °C / S%		0	5	10	15	20	25	30
10	$10^5\Delta_{s,t}$ =	1940	1945	1985	2055	2153	2276	2421
	$-10^3\Delta\gamma_{s,t}$ =	0.7	3.4	5.5	7.2	8.6	9.8	10.7
20	$10^5\Delta_{s,t}$ =	1154	1174	1236	1307	1411	1540	1690
	$-10^3\Delta\gamma_{s,t}$ =	0.4	3.1	5.3	7.0	8.4	9.6	10.5
30	$10^5\Delta_{s,t}$ =	382	416	479	568	680	813	966
	$-10^3\Delta\gamma_{s,t}$ =	0.1	2.9	5.1	6.8	8.2	9.4	10.3
35	$10^5\Delta_{s,t}$ =	0	40	109	202	317	452	607
	$-10^3\Delta\gamma_{s,t}$ =	0	2.8	5.0	6.7	8.0	9.3	10.2

$\triangle_{s,t}$ for a constant value of salinity along each curve as shown in Fig. 1. Note that $10^3 \triangle \Upsilon_{s,t}$ takes negative values from zero to −11.

Select an isohaline curve. Let the value of $10^5 \triangle_{s,t}$ be 'a' where the value of $10^3 \triangle \Upsilon_{s,t} = -11$ and from this value of a, substract the value of $10^5 \triangle_{s,t}$ at which $\triangle \Upsilon_{s,t} = 0$. Let the resultant value be called A.

$$\text{Thus} \quad a = 10^5{}_{s,t} \; (\text{at } 10^3 \triangle \Upsilon_{s,t} = -11) \tag{5}$$

$$\text{and} \quad A = \alpha - 10^5 \triangle_{s,t} \; (\text{at} \triangle \Upsilon_{s,t} = 0) \tag{6}$$

Table II gives the values of 'a' and A at different salinity values. Examination of the nature of each curve which carried a constant value of salinity

Table II. The values of 'a' and 'A' at different salinities.

S‰	a	A
0	3160	432
5	2820	484
10	2490	536
15	2150	590
20	1810	642
25	1470	694
30	1140	746
35	800	800

indicates that $\alpha - 10^3 \triangle_{s,t}$ is exponentially related with $11 - (-10^3 \triangle \Upsilon_{s,t})$ with a as origin and with A as saturation value at which the curve practically tends to be parallel to $\triangle \Upsilon_{s,t}$ axis. Therefore, the possible relation is

$$(\alpha - 10^5 \triangle_{s,t}) = A \left[1 - e^{-\beta (11 + 10^3 \triangle \Upsilon_{s,t})} \right] \tag{7}$$

Where β is constant. From Table II the relation of a and S and that of A and S are found to be linear and they are

$$a = 3150 - 67.5 \;\; S \tag{8}$$

$$A = 432 + 10.5 \;\; S \tag{9}$$

Substituting the values of a and A in terms of S, eq. 7 becomes

$$3150 - 67.5 \; S - 10^5 \triangle_{s,t} = (432 + 10.5 \; S) \left[1 - e^{-\beta (11 + 10^3 \triangle \Upsilon_{s,t})} \right] \tag{10}$$

Applying the condition that both the anomalies of specific volume and viscosity are zero at S = 35‰ and t = 0°C, the above equation reduces to

$$787.5 = 799.5 \; (1 - e^{-11\beta}) \tag{11}$$

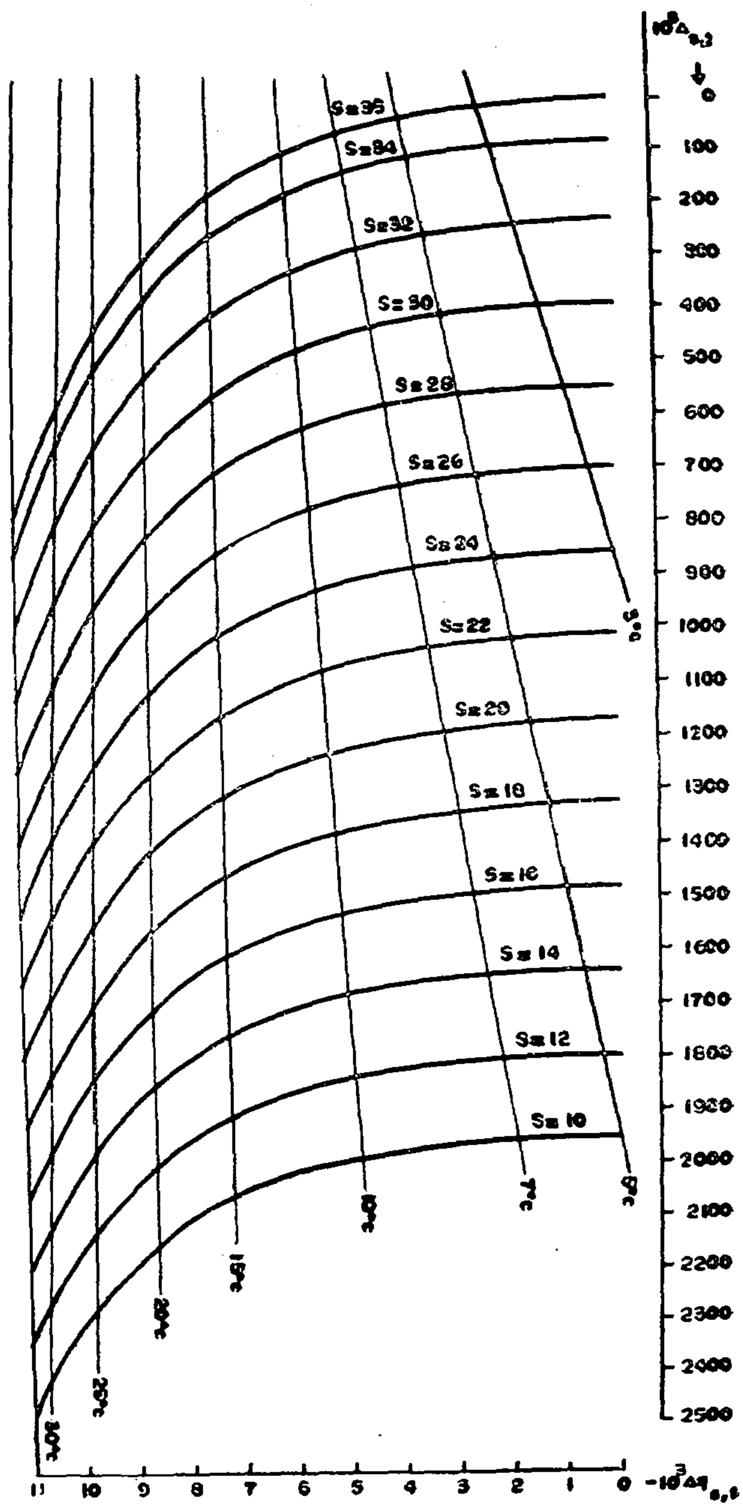

Fig. 1. Viscosity – specific volume anomalies nomogram.

from which

$$\beta = 0.3817 \approx 0.382 \qquad (12)$$

Therefore eq. 10 becomes

$$3150-67.5\ S-10^5\Delta_{s,t} = (432+10.5\ S)\left[1-e^{-0.382\,(11+10^3\Delta\gamma_{s,t})}\right] \qquad (13)$$

A series of curves of viscosity — specific volume anomalies at different fixed values of salinity i.e. each curve carrying a fixed salinity value, can be drawn in the nomogram (Fig. 1) by choosing different salinities in succession and inserting those values in eq. 13. Only a limited number of isohalines are drawn in the nomogram in order to avoid clumsiness, as the size of the diagram is very small. In preparing the actual nomogram of usual size (60 cm x 40 cm) the interval of isohalines can be narrowed down.

At this stage it is very easy to incorporate the set of isotherms at regular intervals in the nomogram. For a set of salinity values whose isohalines are drawn in the nomogram, read the σ_o values corresponding to each salinity value from Knudsen tables. Choosing a convenient fixed temperature value, obtain the set of σ_t values corresponding to σ_o values using the table for σ_t from σ_o and t values. Determine $\Delta_{s,t}$ values corresponding to σ_t values from eq. 3. Thus we arrive at a set of salinity and $\Delta_{s,t}$ values paired with each other for the same fixed temperature value and the line of isotherm is drawn by plotting these paired values of S and $\Delta_{s,t}$. The procedure is repeated for the entire family of isotherms to be drawn in the nomogram. Thus in the nomogram of anomalies of specific volume and viscosity, the two families of curves belonging to temperature and salinity are presented within the frame of viscosity-specific volume, having their anomalies as the rectangular coordinates of the frame of reference.

Usefulness of the nomogram

The characteristic features of temperature-salinity distributions can be represented in the diagram. As mixing takes place between two water types along σ_t line (Newman and Pierson. 1966) and as σ_t is a single valued function of $\Delta_{s,t}$ (eq. 3), therefore mixing takes place along the horizontal line ($\Delta_{s,t} =$ const line) which is equivalent to σ_t line. If a temperature-salinity relation curve is plotted in the diagram tagged with depth values along it, a rising curve indicates more and more stability. If the line is vertically upword, such water body is internally the most stables. As the axis of viscosity anomaly carried negative values, the more the progress of the curve towards right the more viscous the waters would be, which therefore reveals the dynamic nature of the water disregarding the current (flow) system. This is because the dynamic viscosity, unlike eddy viscosity, is independent of the state of motion of the fluid and is only a characteristic property of the fluid comparable with the elasticity of a solid body (Newman and Searle, 1951; Sverdrup, Johnson & Fleming, 1961). Therefore the T-S relation curve in the nomogram of viscosity-specific volume anomalies reveals more details of physical and dynamic characteristics of the water masses than the conventional T-S diagram. As sonic waves at higher range of audible frequency are damped by viscosity (Graff, 1981), the diagram may be useful for sound attenuation studies in sea water as well.

The lines of each family of curves can be drawn densely and perfectly with the aid of a good computer.

It is hoped that the viscosity-specific volume anomalies nomogram would serve a useful purpose in the various fields connected with oceanic studies. If it happens, that would serve a good tribute to Dr. V.V.R. Varadachari who inspired me to understand the intricacies of the subject of physical oceanography.

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